

Klaus Solbach, Ingo Wolff
University of Duisburg
Department of Electrical Engineering
Bismarckstrasse 81
41 Duisburg 1, West Germany

Abstract

A method is described for the exact calculation of the field distributions and the phase constants of single and coupled dielectric image lines. The theoretical results have been proved by experiments with dielectric image lines fabricated of casting resins.

Introduction

Dielectric image lines are used as a basic of integrated millimeter-wave circuits; it is hoped that they will solve the problems which are known in connection with the application of microstrip lines in the millimeter-wave range. Therefore more attention has been paid to this kind of microwave guides in the last years by several authors; furthermore dielectric waveguides have been proposed for application in the optical range. Papers of Goell¹ and Marcatili² which base on investigations of Schlosser and Unger³ shall be mentioned here.

In this paper an exact solution is presented for the calculation of the phase constant and the field distributions of one single or two coupled dielectric image lines of rectangular cross section. The method presented can be used directly for the calculation of the dielectric image line; it avoids the approximations made by Marcatili. Furthermore it gives the possibility to discuss the physical structure of the electromagnetic waves on single and coupled image lines using the calculated field distributions and it gives the possibility to compute the exact attenuation constant due to dielectric losses in the lines.

The Calculation Method

In Fig. 1 a the investigated waveguide is shown; it consists of two coupled image lines of rectangular cross section. The two lines of width $2w$ and height h are separated by a distance c ; they are mounted on a metal ground plane. To define a proper eigenvalue problem a second metal plate of infinite conductivity is placed in a distance d parallel to the ground plane. The influence of this plate on the properties of the lines can be omitted by choosing a large distance d . Because of the symmetry of the structure an even and an odd mode can propagate on the coupled lines. The symmetry plane between the lines is a magnetic wall in the case of the even mode and an electric wall in the case of an odd mode; it means that the structure to be calculated can be reduced to that shown in Fig. 1 b, where the plane $x=b$ is a magnetic wall in the case of the even mode and an electric wall in the case of the odd mode. If the properties of the single, uncoupled line shall be calculated the limitation $b \rightarrow \infty$ is considered. Under this condition the two lines shown in Fig. 1a no longer influence each other. For the calculation of the electromagnetic fields, which are possible on a line as shown in Fig. 1 b, the field region is subdivided into four partial regions (I-IV) and a complete set of field solutions is derived for

each subarea. It is assumed that the dependence on the z -coordinate can be described by an exponential function $\exp(-j\beta z)$, β being the phase constant. The x - and y -dependences of the fields in the regions I, III and IV are formulated using harmonic functions of TE^y - and TM^y -kind, so that the boundary conditions are fulfilled on the defined boundaries. In region I it is assumed that the x -dependence of the fields can be given by an exponential decay whereas the y -dependences are described by a harmonic function again.

The boundary conditions in the plane $y=h$, ($-w \leq x \leq +w$) between regions II and III can be fulfilled independently of the remaining conditions. Furthermore it is possible to match the fields of the TE^y -modes and the TM^y -modes as well as the fields which are even and odd corresponding to the plane $x=0$ separately on this boundary. If the boundary condition at $x=-w$ ($0 \leq y \leq d$) and $x=+w$ ($0 \leq y \leq d$) shall be fulfilled, all modes have to be taken into account, and an infinite set of equations results for the unknown amplitudes of the field potentials in the four subareas. If the phase constants of the waves on the dielectric image line shall be computed, the zeros of the determinant of the set of equations must be determined. Using the phase constant the field potentials for the four field regions can be derived. From the field potentials the field distributions of the electric and magnetic fields can be calculated.

Numerical Results

The possible modes on the dielectric image lines are described in a way similar to that of other waveguides. In² and⁴ it is assumed that the E_y - and H_x -components are much larger than all other field components. Indices p and q are introduced which are equal to the number of maxima of the E_y -component in x - and y -direction respectively. The assumption that the E_y -component is large compared to all other components of the electric field strength was the basis of the approximation made by Marcatili.

As can be shown by means of the theory presented here this assumption in general is not valid. The assumption only is applicable to the fundamental modes EH_{11} , or the EH_{11e} and EH_{11o} modes in the case of coupled lines, on low permittivity lines ($\epsilon_r < 3$) and arbitrary aspect ratio w/h , see Figs. 2, 3. High permittivity lines of very low or very high aspect ratio w/h

(degenerated into dielectric slab or dielectric sheet waveguide) also can be described well under that assumption. In all the remaining cases of modes, especially for high permittivity lines, there is no predominance of a single field component, see Fig.4.

Confirming to the degree of justification of Marcattili's assumption his theory⁴ as well as the modified theory due to Toullos and Knox⁵ yield propagation constants different to those resulting from the theory presented here (see Figs. 5, 6), as well as to Goell's¹ results for single lines.

Furthermore it can be shown that the approximate theory of Marcattili does not give the complete set of modes. For example the EH_{21} mode (see Fig.7) which can be calculated by the theory presented and which has been proven experimentally cannot be computed using Marcattili's approximation.

Experimental Results

The theoretical results have been proven experimentally in the millimeter-wave range (26GHz-40GHz) using dielectric image lines fabricated from paraffin wax and casting resins in a die-casting process.

The production-technique along with results from measurements of the field distributions of single and coupled image lines already have been published elsewhere⁵.

Especially the agreement between measured phase constants and the results of the theory presented here has been found to be very good, see Fig. 7.

References

1. J.E. Goell, "A circular-harmonic computer analysis of rectangular dielectric waveguides", Bell Syst. Tech.J., Vol.48, pp.2133-2160, Sept.1969.
2. E.A.J. Marcattili, "Dielectric rectangular waveguide and directional coupler for integrated optics", Bell Syst.Tech.J., Vol.48, pp.2071-2102, Sept.1969.
3. W. Schlosser, H.-G. Unger, "Partially filled waveguides and surface waveguides of rectangular cross section", Advances of Microwaves, Vol. 1, pp.319-387, Academic Press, New York, 1966.
4. R.M. Knox, P.P. Toullos, "Integrated circuits for the millimeter through optical frequency range", Symposium on Submillimeterwaves, pp.497-516, Polytechnic Press, New York, 1970.
5. K. Solbach, "The fabrication of dielectric image lines using casting resins and the properties of the lines in the millimeter-wave range", IEEE-Trans. MTT-24, No. 11, November 1976.

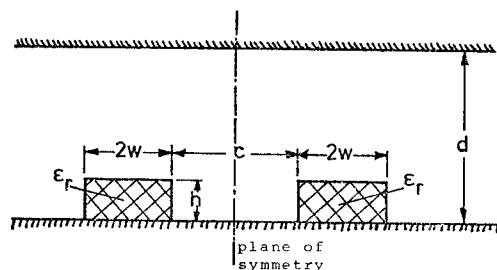


Fig. 1a-The cross section of the investigated coupled dielectric image lines.

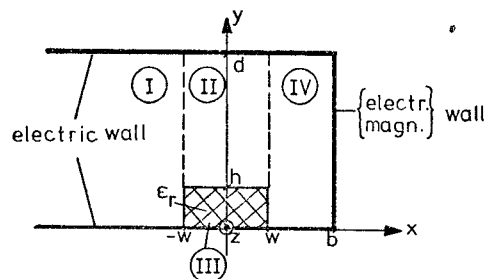


Fig. 1b-The cross sectional structure adopted for the calculation of phase constant and field distribution of coupled dielectric image lines.

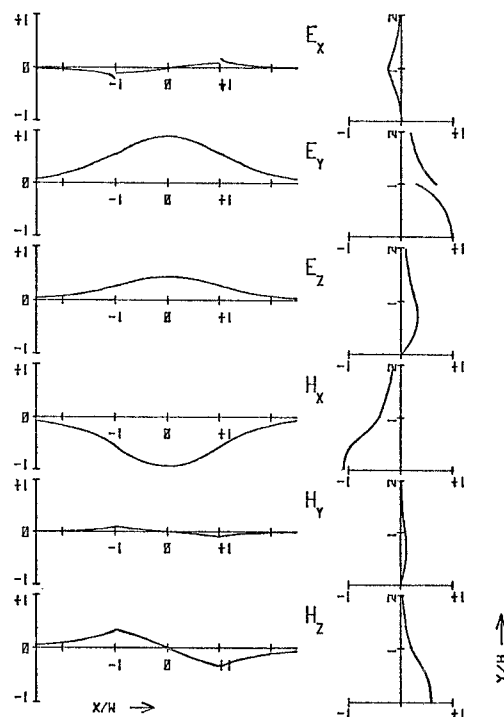


Fig. 2-Normalized calculated field distributions of the EH_{11} mode on a dielectric image line in a horizontal plane at $y=0.8h$ and in a vertical plane at $x=0.9w$. $w/h=1$, $b/w \rightarrow \infty$, $d/h \rightarrow \infty$, $B=1.2$, $B/B_0 = 1.1854$, $\epsilon_r=2.22$, where B = normalized frequency $= \frac{4h\sqrt{\epsilon_r-1}}{\lambda_0}$.

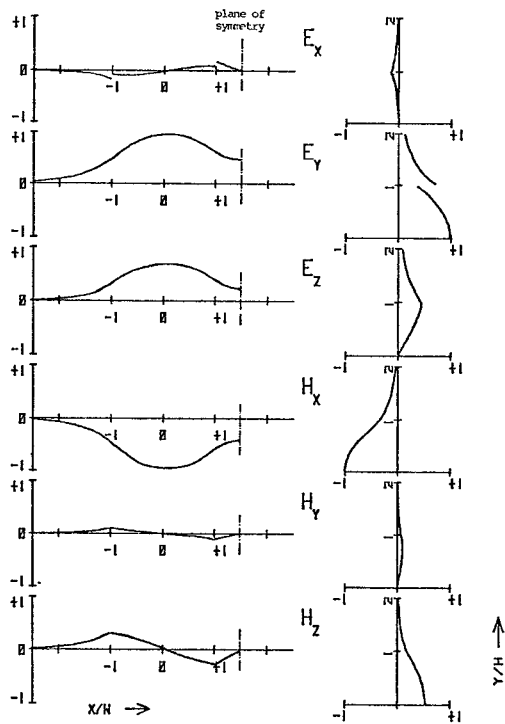


Fig. 3-Normalized calculated field distributions of the EH_{11e} mode on coupled dielectric image lines in a horizontal plane at $y=0.8h$ and in a vertical plane at $x=0.9w$. $w/h=1$, $b/w=1.5$, $d/h \rightarrow \infty$, $B=1.5$, $\beta/\beta_0=1.2756$, $\epsilon_r=2.22$.

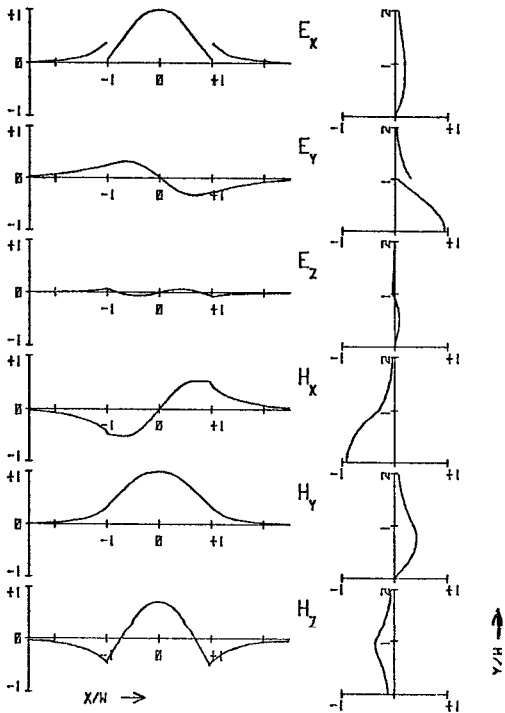


Fig. 4-Normalized calculated field distributions of the HE_{21} mode on a high permittivity dielectric image line in a horizontal plane at $y=0.8h$ and in a vertical plane at $x=0.9w$. $w/h=1$, $b/w \rightarrow \infty$, $d/h \rightarrow \infty$, $B=2.0$, $\beta/\beta_0=1.9183$, $\epsilon_r=10$.

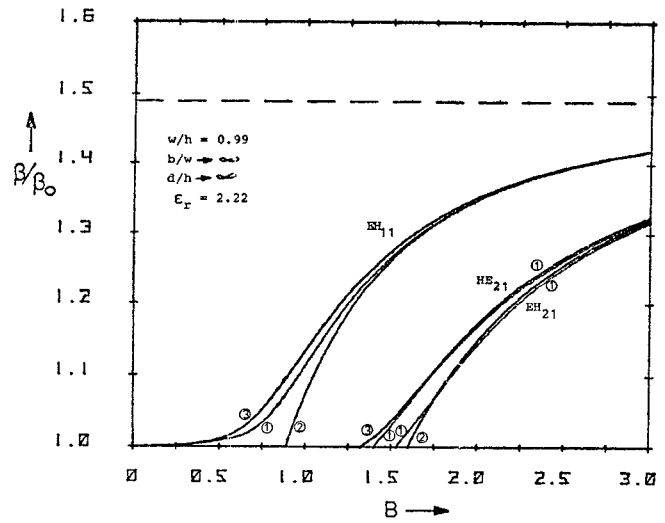


Fig. 5-Normalized phase constant of a dielectric image line versus the normalized frequency B .
① This theory, ② Marcattili's² approximation, ③ approximation of Toullos and Knox.

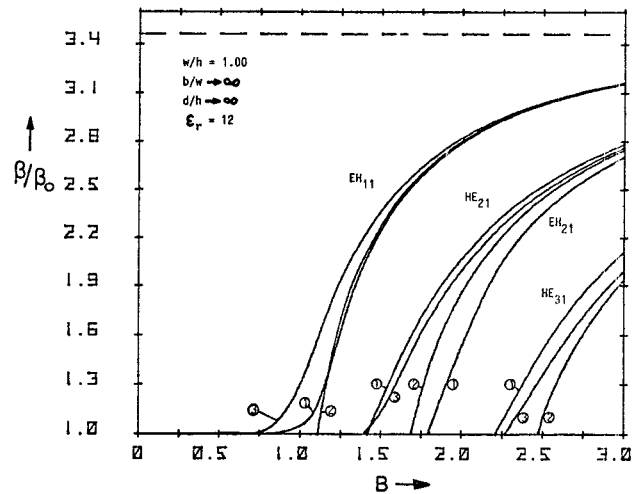


Fig. 6-Normalized phase constant of a high permittivity dielectric image line versus the normalized frequency B .
① This theory, ② Marcattili's² approximation, ③ approximation of Toullos and Knox.

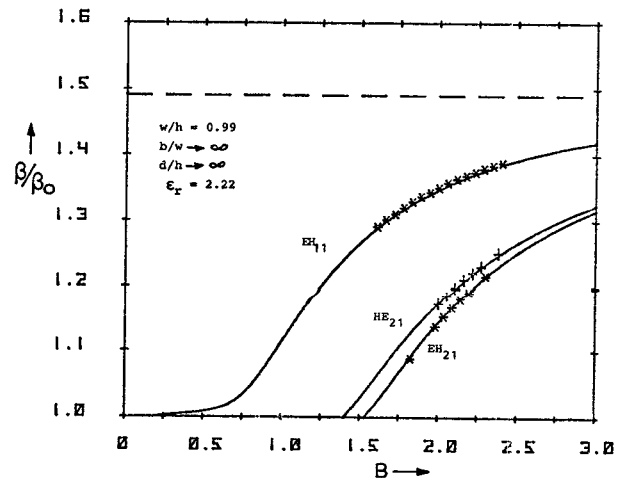


Fig. 7-Normalized phase constant of a dielectric image line versus the normalized frequency B .
— This theory, + and * experimental data from lines of height $h=4.1\text{mm}$.